Validation of Implicit Algorithms for Unsteady Flows Including Moving and Deforming Grids

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MECHANICAL ENGINEERING

Desirable Features in an Unsteady Flow Solver

- Large time steps
  - Quicker turnaround
  - Fewer grid assembly or grid deformation steps per solution
- Temporal accuracy
  - Local convergence at each time step (errors propagate in time)
  - Grid convergence
  - Low numerical dissipation

Time Step Dilemma

- Time scale of physics (micro-milliseconds)
  - Shedding frequency
  - Reaction rates
  - Turbulence scales
- Time scale of problem (seconds)
  - Store drop
  - Aircraft maneuver
  - Engine transient
  - Flutter cycle

Deforming Grids:
Geometric Conservation Law

Geometric Conservation Law (GCL)

Navier-Stokes Equations:

$$\boxed{\mathcal{V}\frac{\partial Q}{\partial t}+Q\frac{\partial \mathcal{V}}{\partial t}}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}+\frac{\partial G}{\partial z}=0$$

Where the conservation variables are defined as:

$$q=Q\mathcal{V}$$

Geometric Conservation Law (GCL)

Relationship between volume and surface area:

$$\frac{d}{dt}\int_{\Omega} d\Omega = -\oint_{\partial\Omega} \vec{c} \bullet \vec{n}\, ds$$

First or second order time implicit algorithm:

$$\frac{(1+\theta_2)\boxed{V^{n+1}}\Delta Q^{n+1} - \theta_2 \boxed{V^{n-1}}\Delta Q^{n}}{\Delta t} + \boxed{Q^n RHS_{GCL}^{n+1}} + RHS^{n+1} = 0$$

Where the GCL term is given by:

$$RHS_{GCL} = \frac{\partial \xi_t}{\partial \xi} + \frac{\partial \eta_t}{\partial \eta} + \frac{\partial \zeta_t}{\partial \zeta}$$

Subiteration Strategy

RHS

$$-\left[\frac{(1+\theta_2)V_0^{n+1}\left(Q_0^{n+1,m}-Q_0^{n}\right)-\theta_2 V_0^{n-1}\Delta Q_0^{n-1}}{\Delta t}+Q_0^{n}RHS_{GCL}^{n+1}+RHS_0^{n+1,m}\right]=$$

$$\left[\frac{V_0^{n+1}I}{\Delta\tau}+\frac{(1+\theta_2)V_0^{n+1}I}{\Delta t}+\frac{\partial RHS_0^{n+1,m}}{\partial Q_0}\right]\Delta Q_0^{n+1,m+1}+\left[\frac{\partial RHS_i^{n+1,m}}{\partial Q_i}\right]\Delta Q_i^{n+1,m+1}$$

Diagonal

Off Diagonal

Newton Method:

$$\Delta\tau=\Delta t$$

$$\frac{V_0^{n+1}I}{\Delta\tau}=0$$

Dual Time Stepping:

- Locally converge inner iteration
- Use convergence acceleration techniques

Flow Solvers Examined

NXAIR Flow Solver

- HLLEm inviscid flux
- 3rd order MUSCL
- van Albada limiters
- 2nd order time with GCL
- Unfactored SSOR
- Newton subiterations
- Turbulence models coupled inside Newton iteration
- Wall functions

OVERFLOW2 Flow Solver

- 2nd or 4th order central or 3rd order Roe inviscid flux
- 1st or 2nd order time
- Pulliam-Steger diagonalized ADI
- Newton or dual time stepping subiterations
- Freestream addition correction

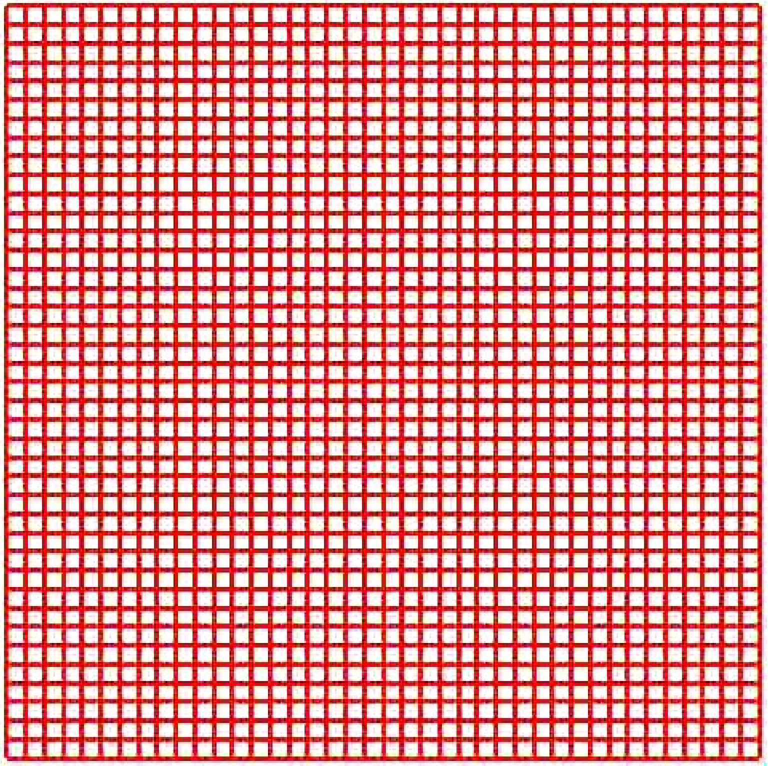
Deforming Body Validation

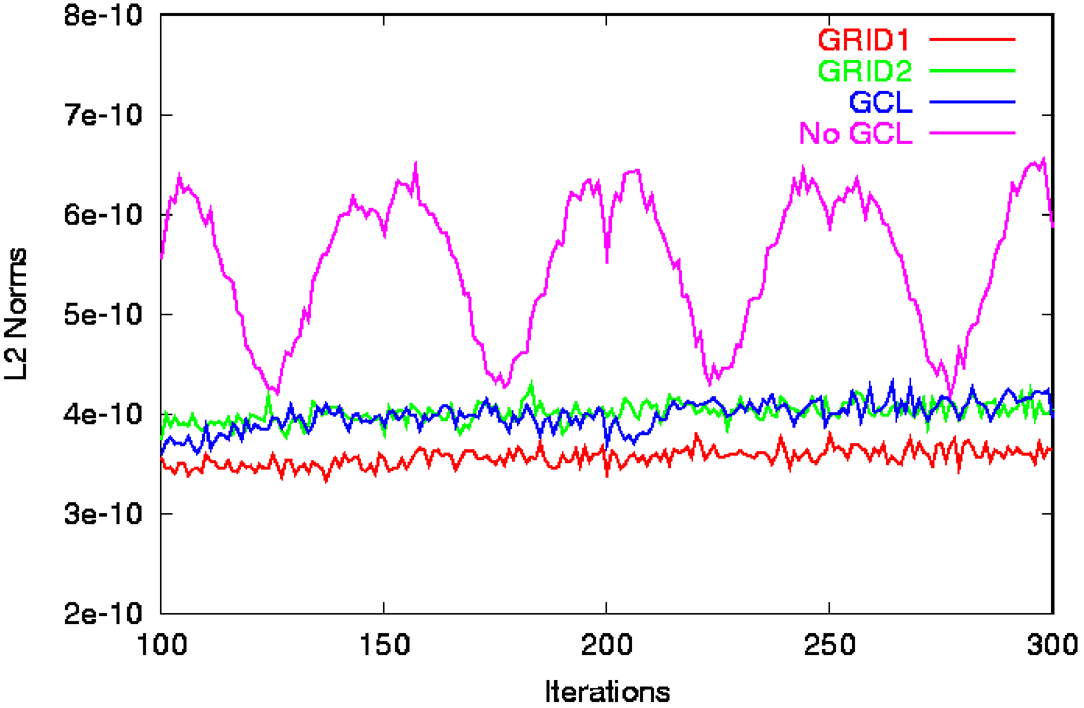
Deforming Box GCL Test

$$x_{comp} = x_1 + f\left(x_2 - x_1\right)$$

$$y_{comp} = y_1 + f\left(y_2 - y_1\right)$$

$$f = \left(\sin\left(\frac{n\pi}{100}\right)\right)^2$$



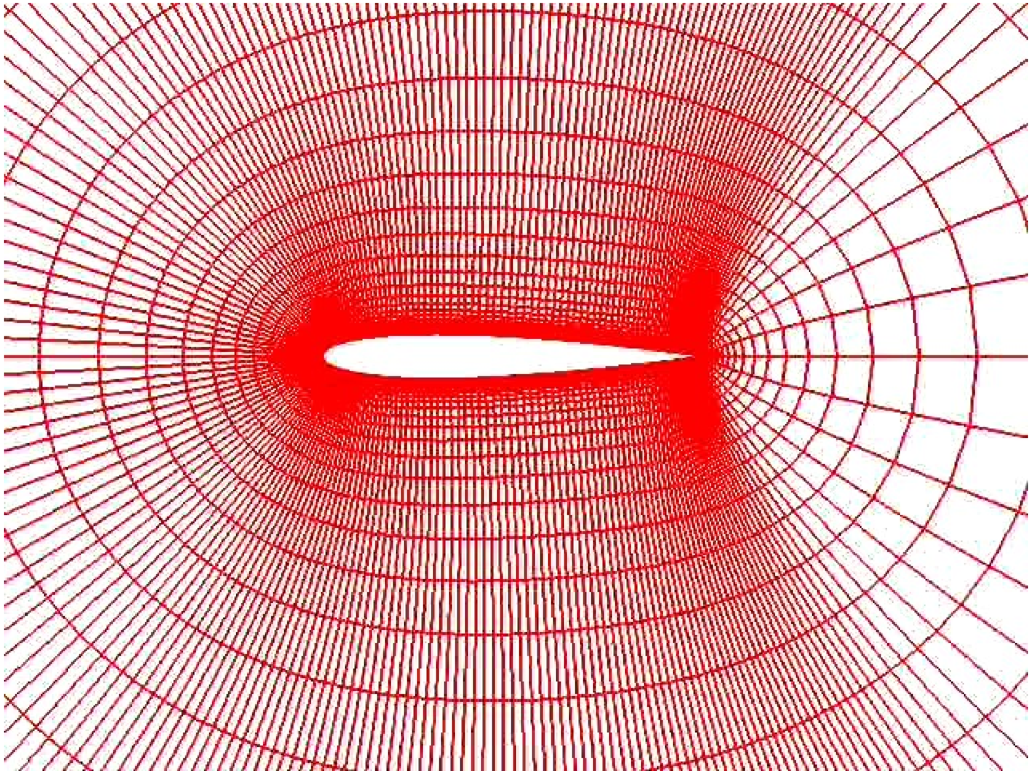


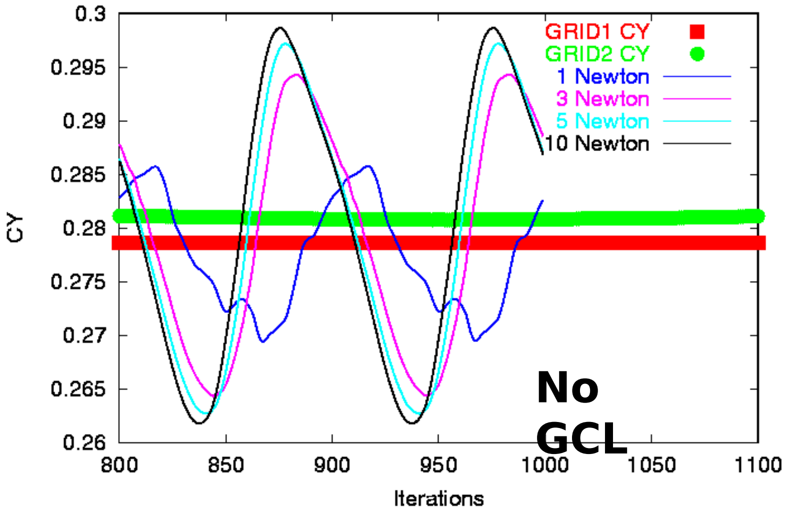
Deforming NACA0012
M=0.5 α=2°

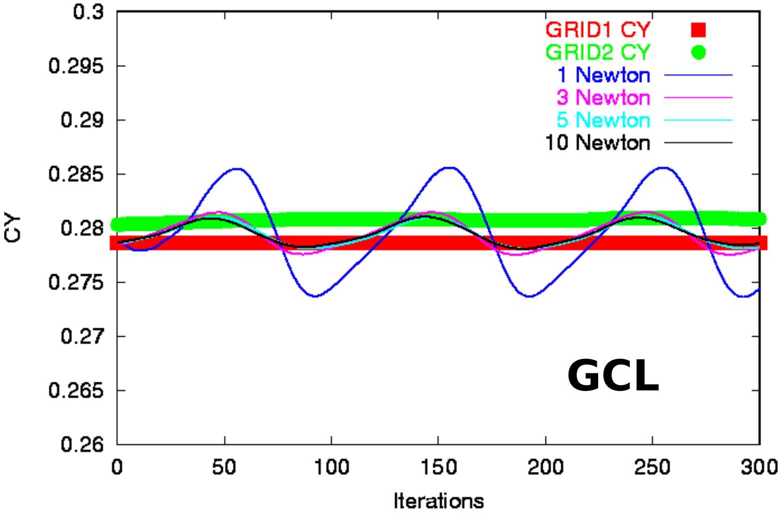
$$x_{comp} = x_1 + f\left(x_2 - x_1\right)$$

$$y_{comp} = y_1 + f\left(y_2 - y_1\right)$$

$$f = \left[\sin\left(\frac{n\pi}{100}\right)\right]^2$$







Unsteady Flow Validation

Test Case Selection Criteria

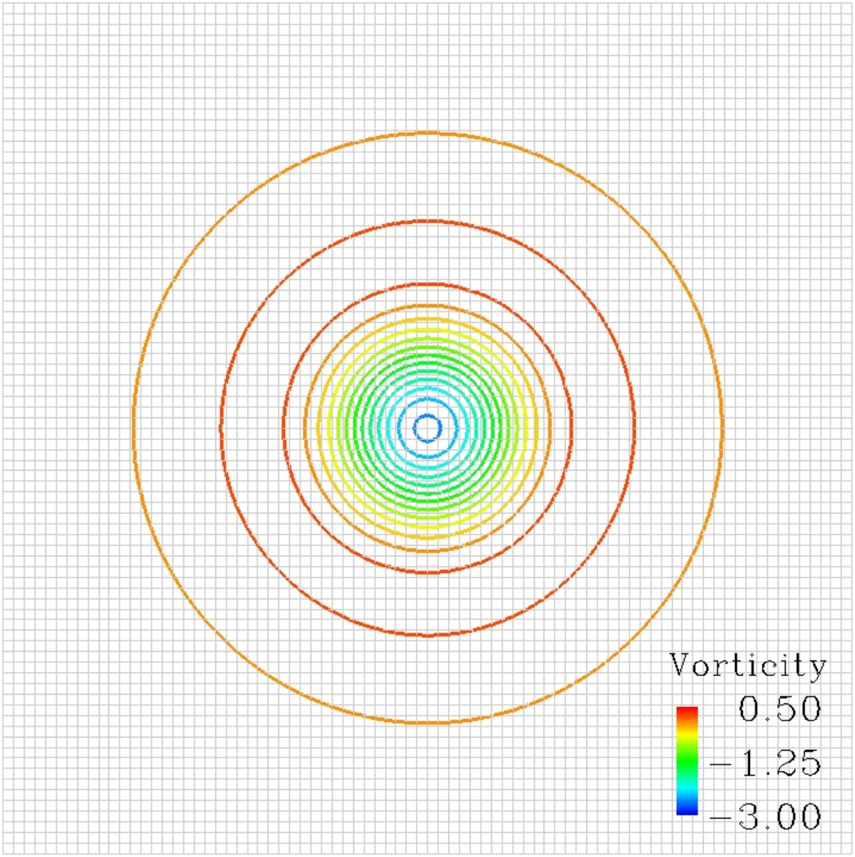
- 2D with simple geometry
- Cases with analytical solutions
  - Inviscid vortex convection
  - Shock tube
- Cases with periodic behavior
  - Laminar cylinder vortex shedding
  - Pitching airfoil with attached boundary layer

Code Evaluation Criteria

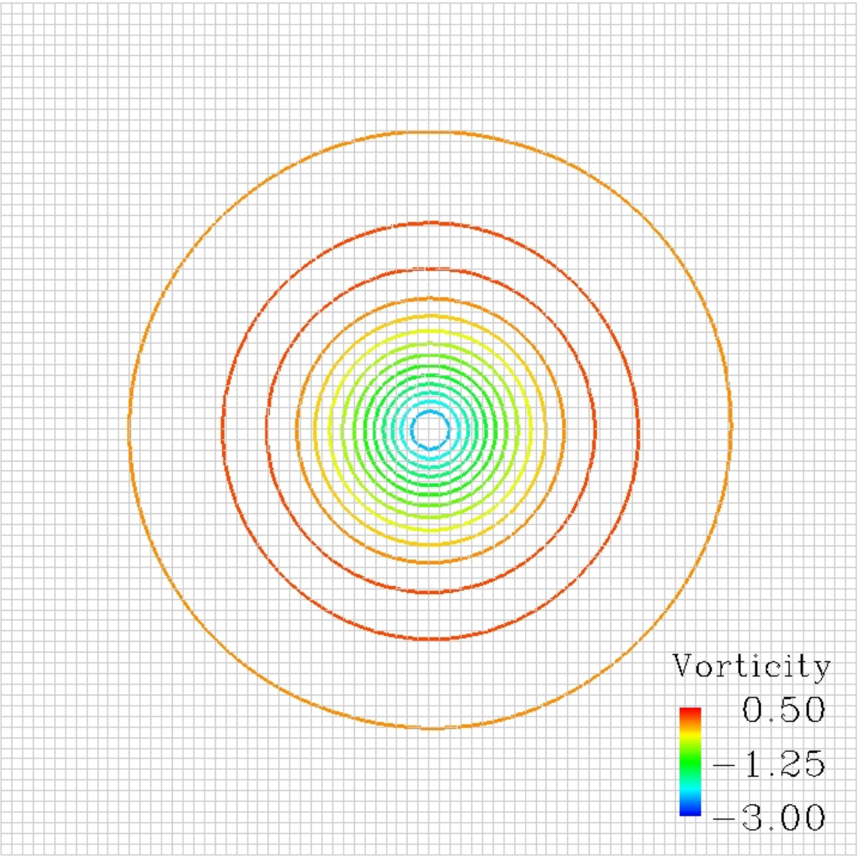
- Local convergence in time
  - Time step variation
  - Subiteration variation
- Ability to capture relevant physics
- Cost of solution

Inviscid Vortex Convection

Initial Vortex



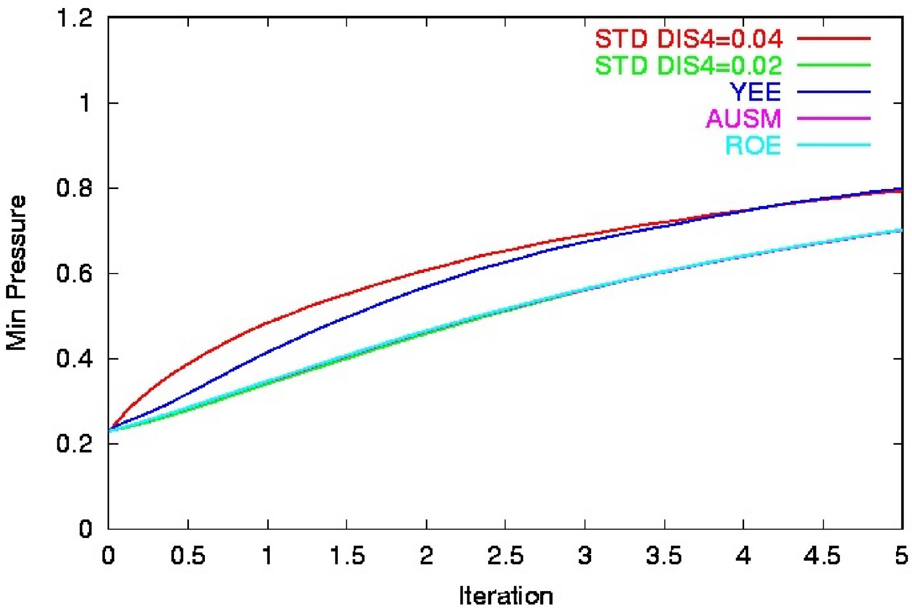
5 Grid Cycles



Inviscid Vortex Convection

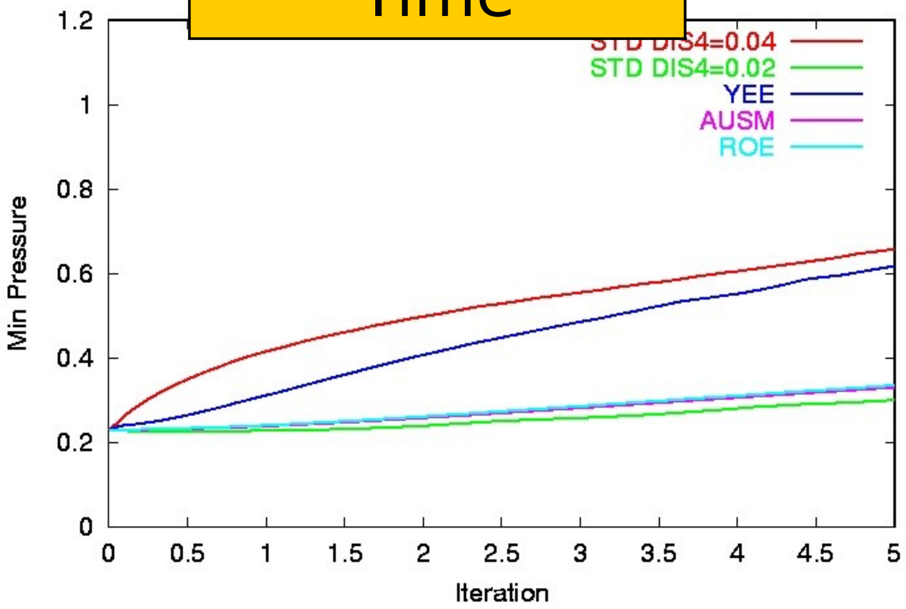
OVERFLOW2

1st Order Time



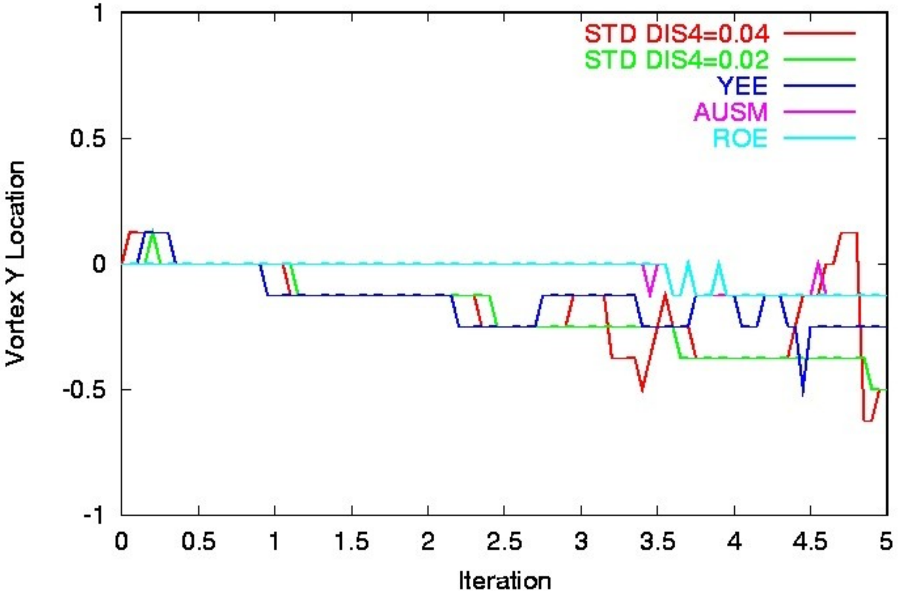
OVERFLOW2

2nd Order Time

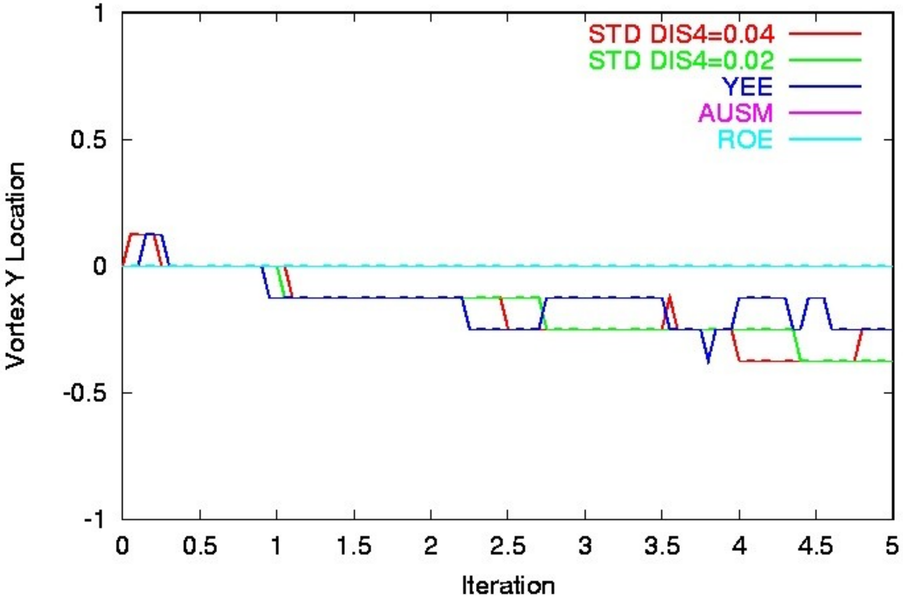


Inviscid Vortex Convection

OVERFLOW2
2nd Order Time
No Newton



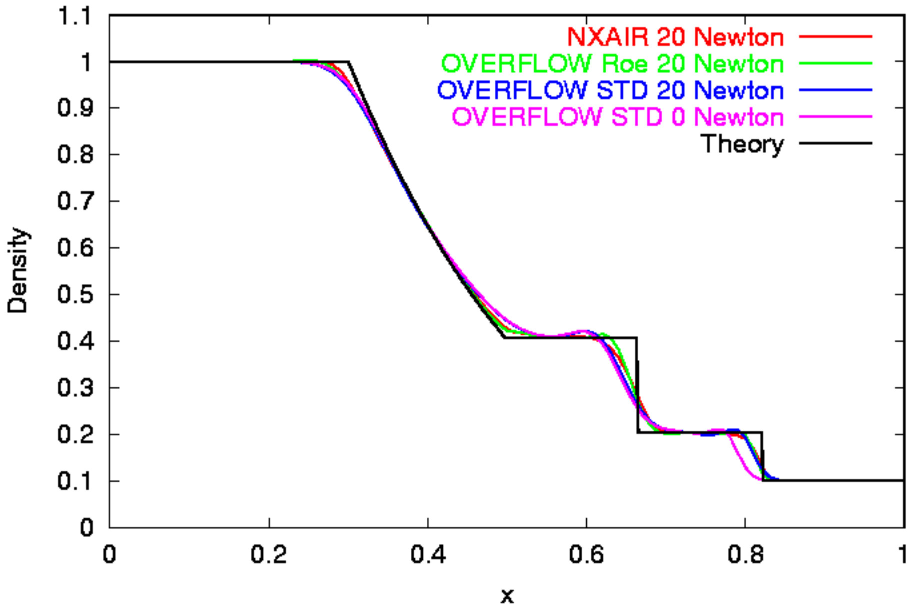
OVERFLOW2
2nd Order Time
3 Newton

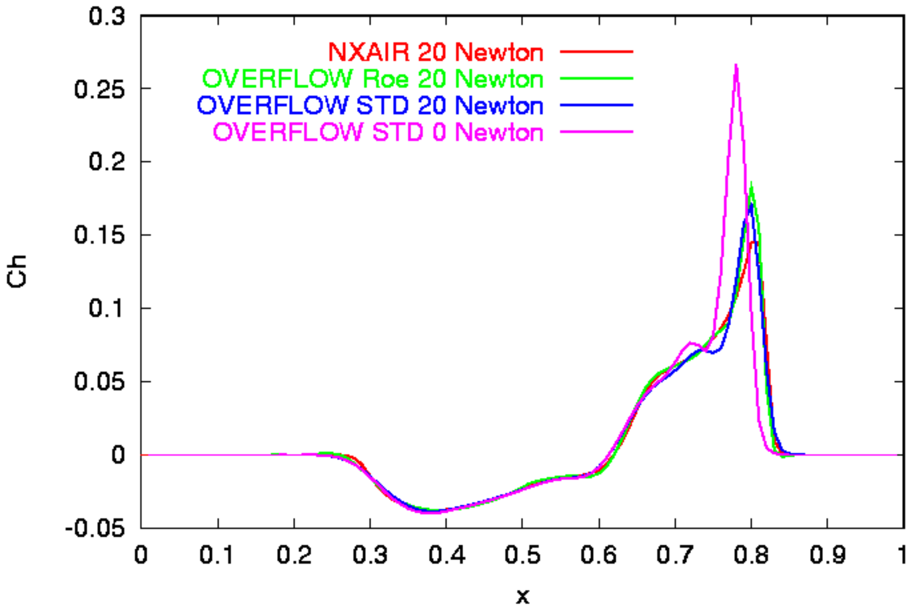


Viscous Shock Tube

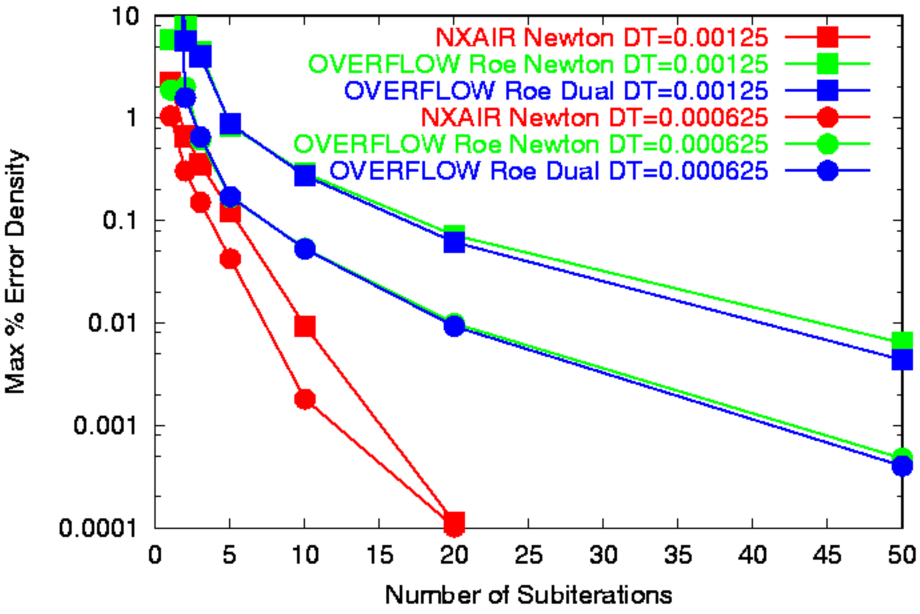
- Left state: ρ/ρ_{ref} = 1.0, p/p_{ref} = 1.0, T_{wall}/T_{ref} = 1.0
- Right state: ρ/ρ_{ref} = 0.1, p/p_{ref} = 0.1, T_{wall}/T_{ref} = 1.0
- Re (based on a_{inf}) = $1.0\text{x}10^5$
- Nondimensional time step = 0.00125
- Results evaluated at nondimensional time = 0.2 (160 iterations)

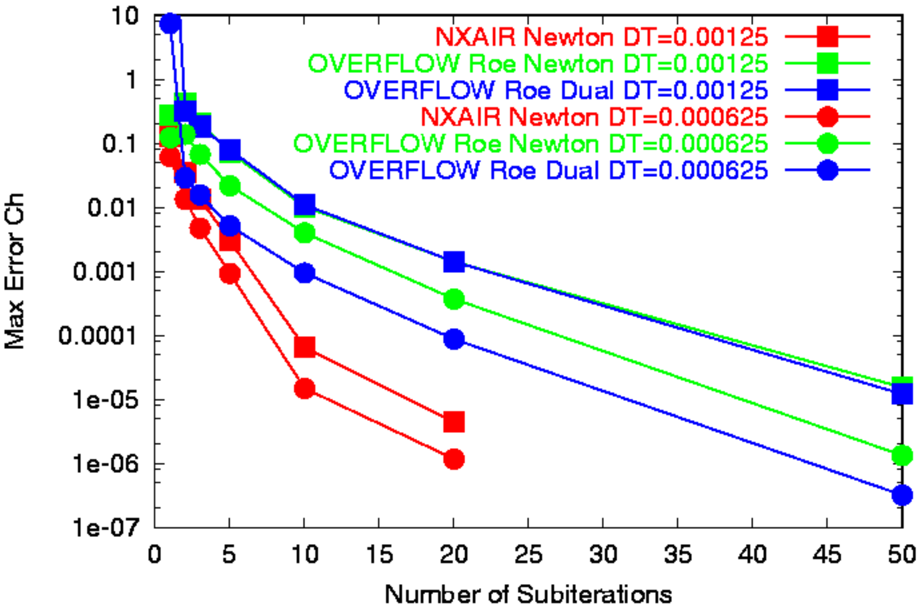
Viscous Shock Tube



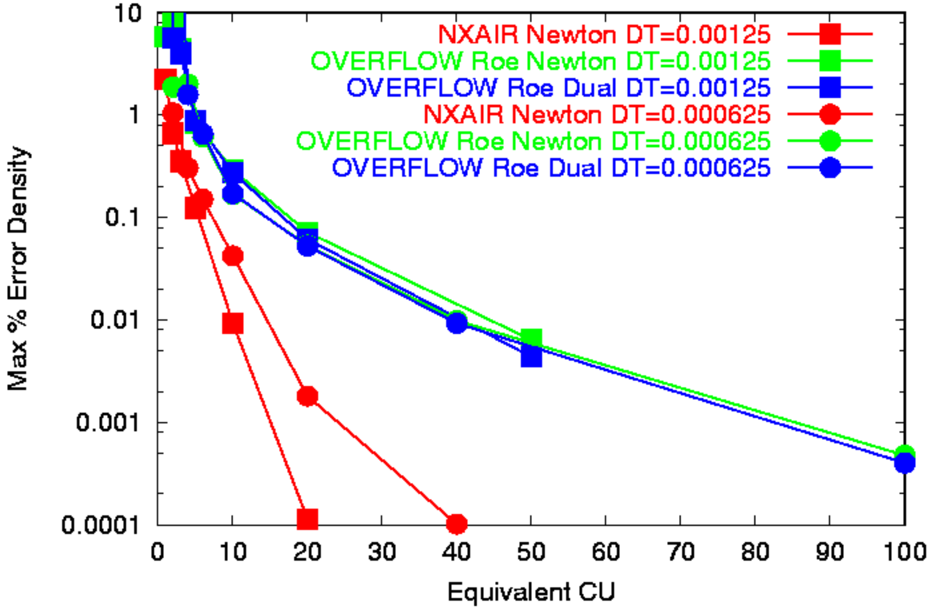


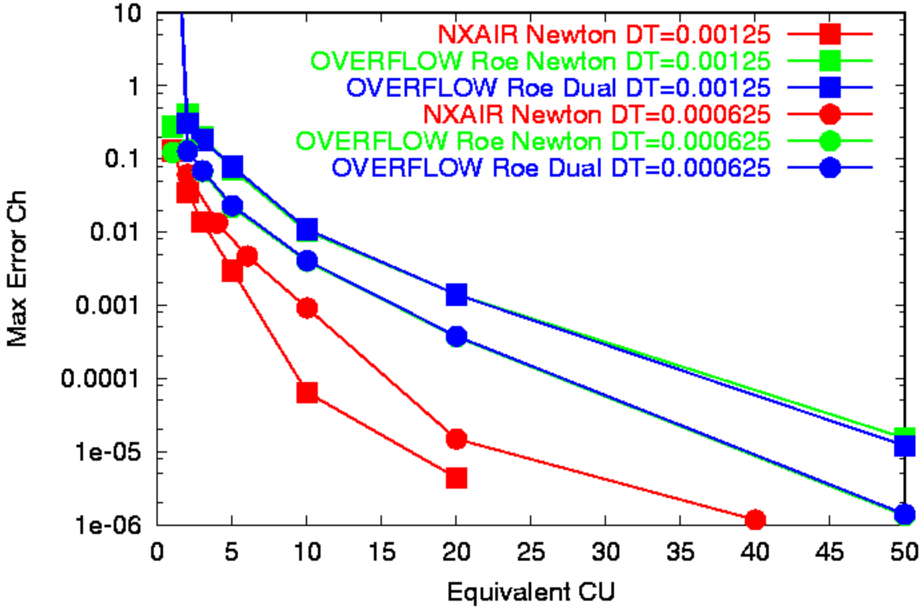
Viscous Shock Tube Subiteration Convergence





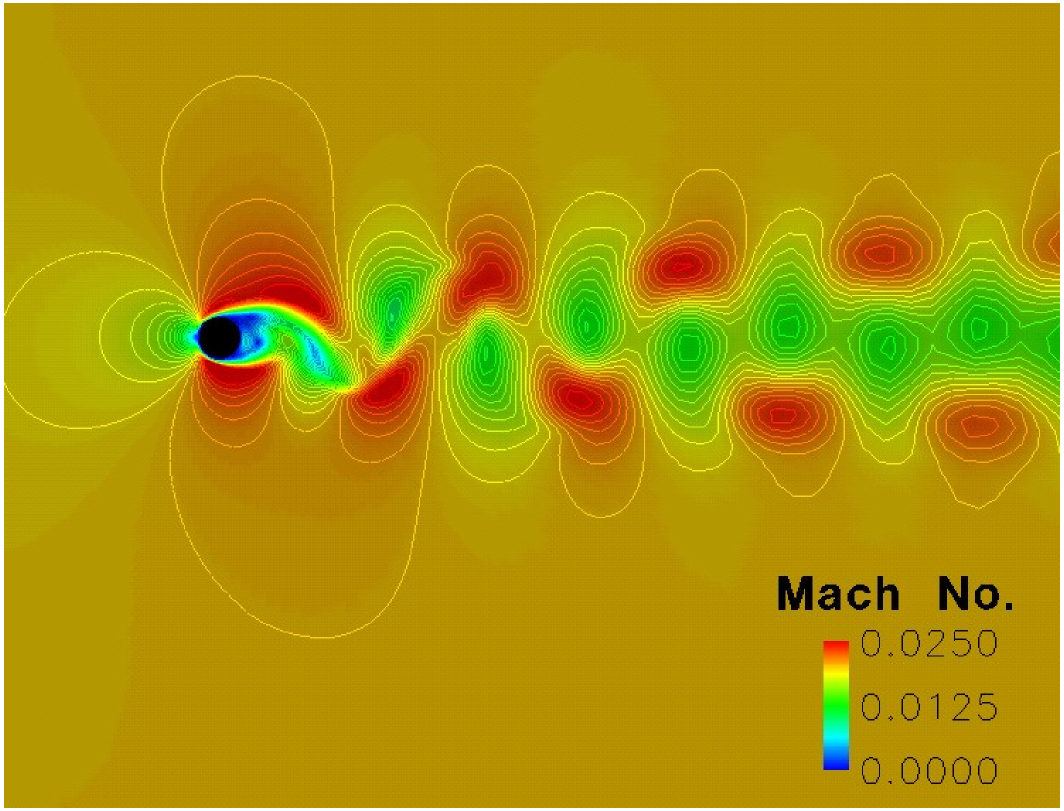
Viscous Shock Tube Computational Cost



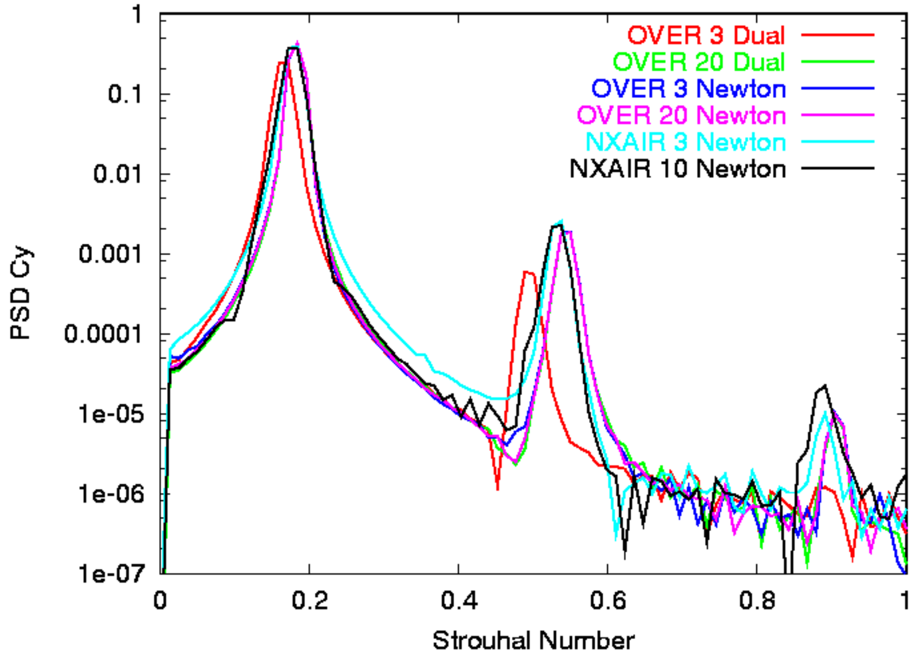


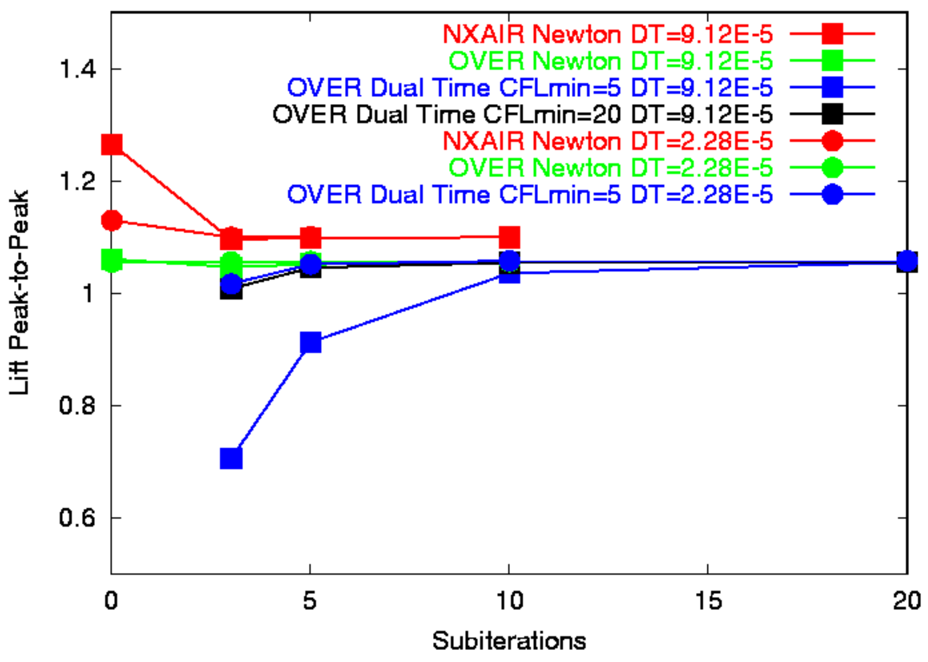
Laminar Cylinder in Crossflow

- $M=0.2$
- $Re_D=150$
- $\Delta t = 9.12\times10^{-5}$ sec. (CFL=92)
- $\Delta t = 2.28\times10^{-5}$ sec. (CFL=23)
- 401x201 grid

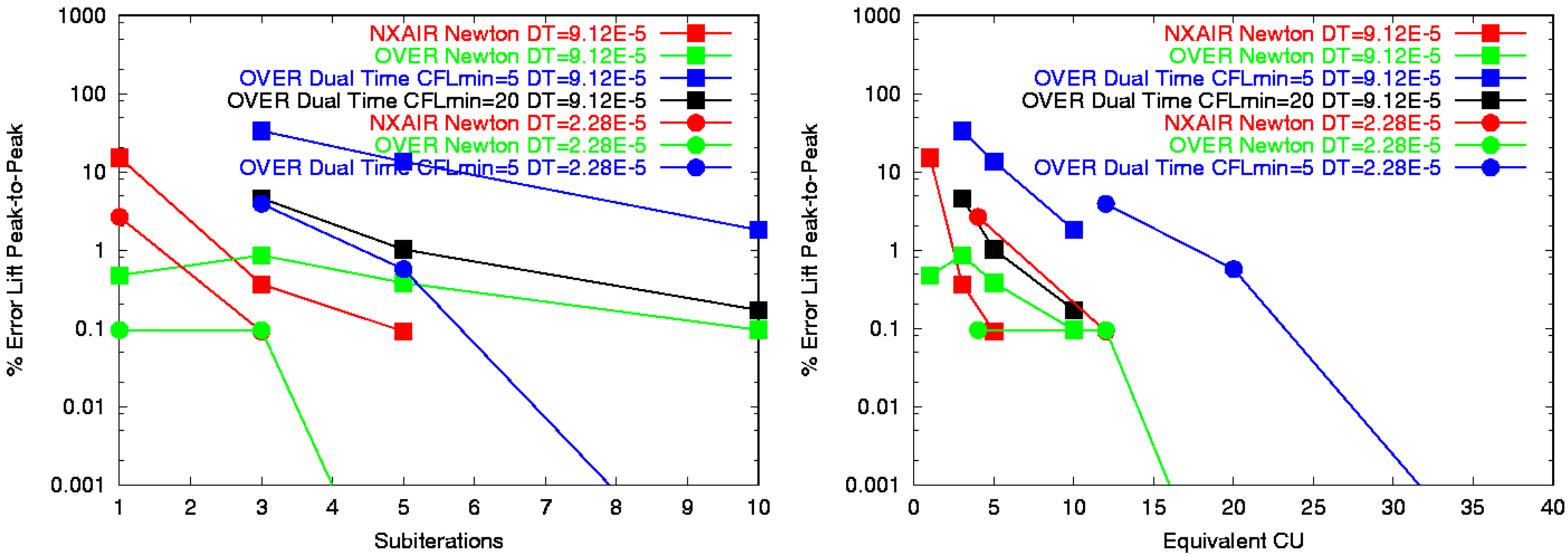


Laminar Cylinder in Crossflow





Laminar Cylinder in Crossflow Error and Cost



Moving Body Validation

Pitching NACA0015 Airfoil

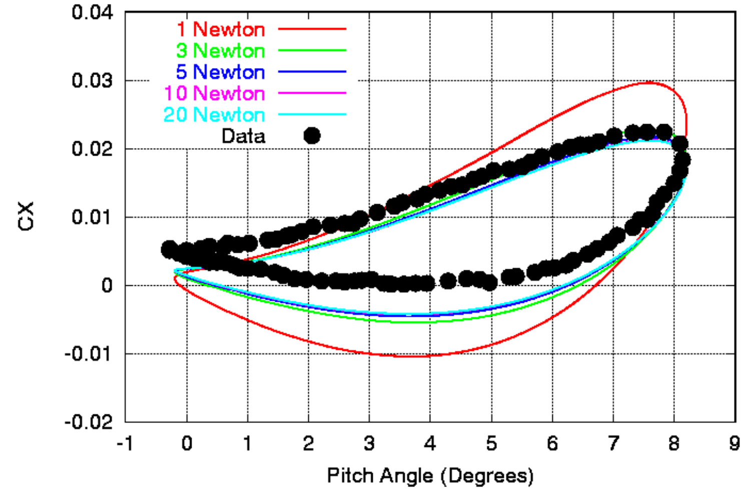
$M=0.29 \quad Re_c=1.95\times10^6$

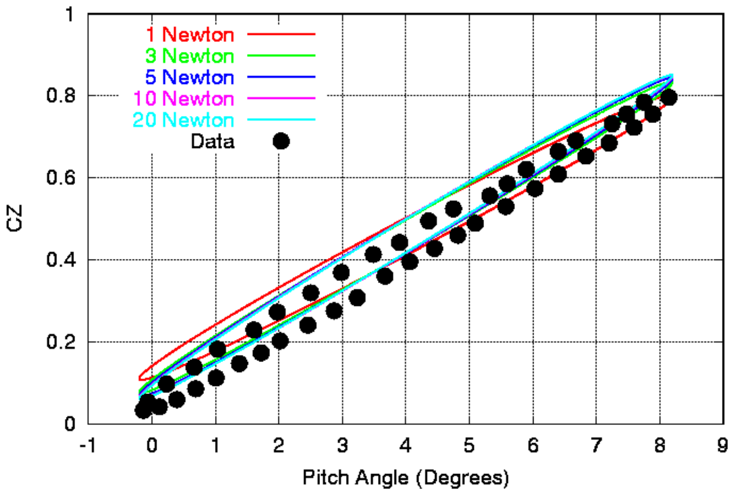
$f=10$ hz

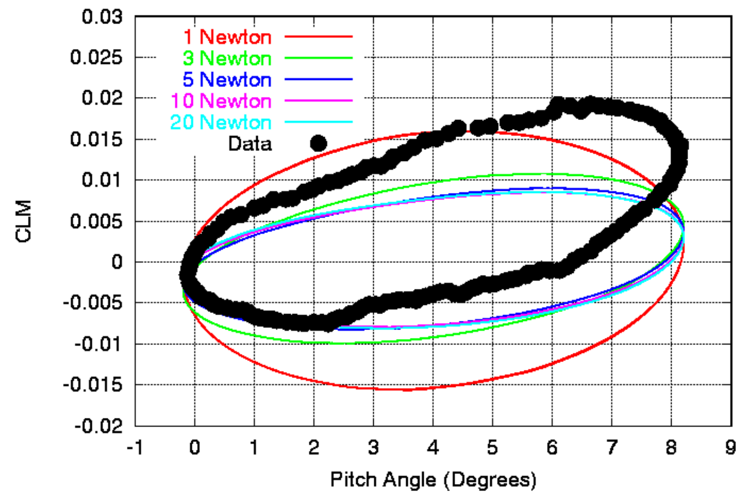
$\Delta t=1/5120$ and $1/10240$ sec.

441x71 grid

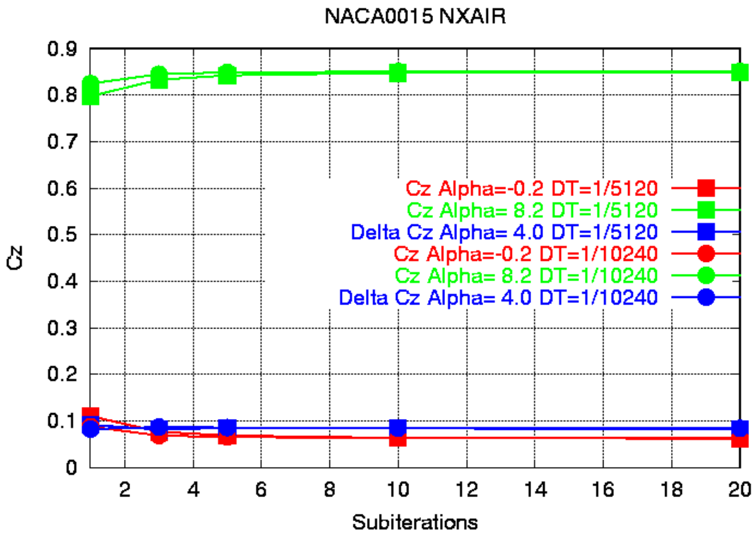
$\alpha = 4^0 + 4.2^0 \sin(2\pi f t)$

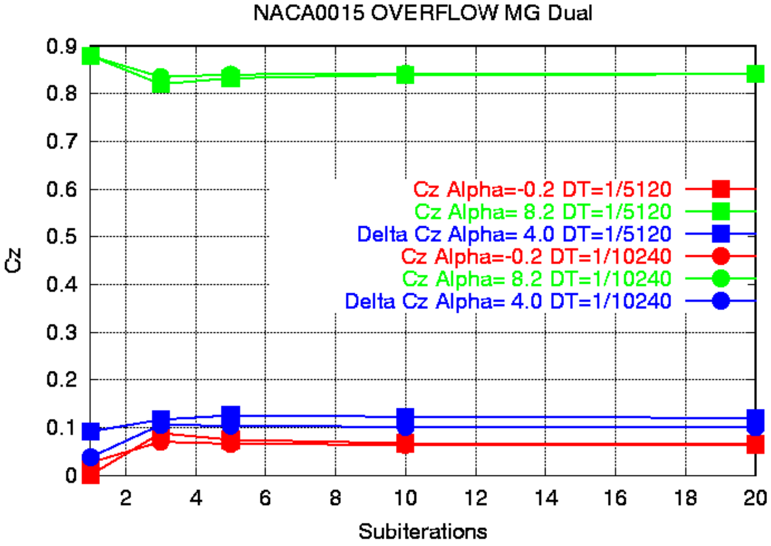


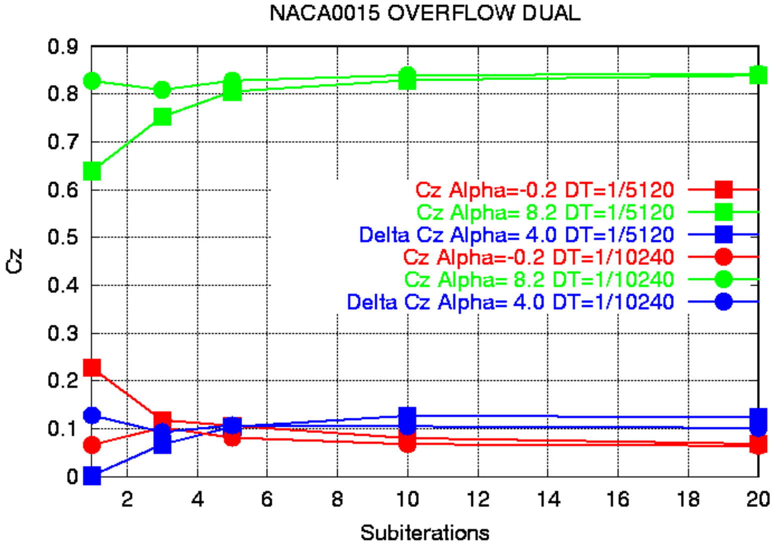


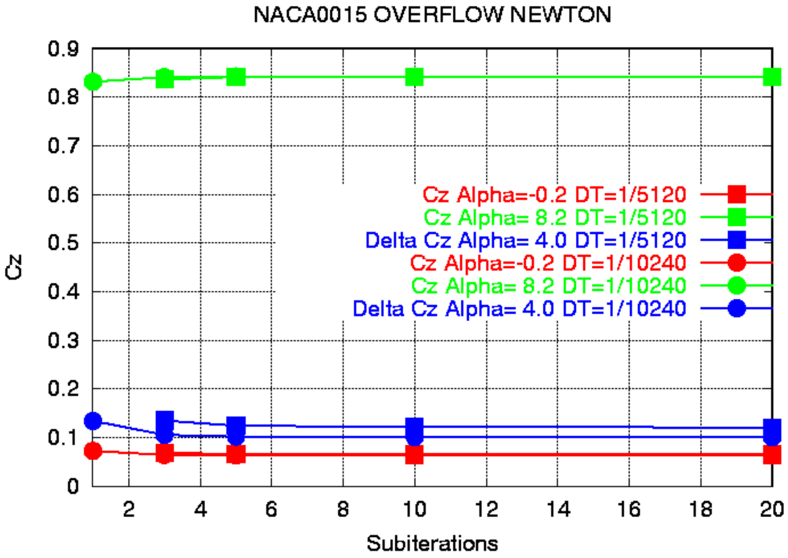


Pitching NACA0015 Airfoil

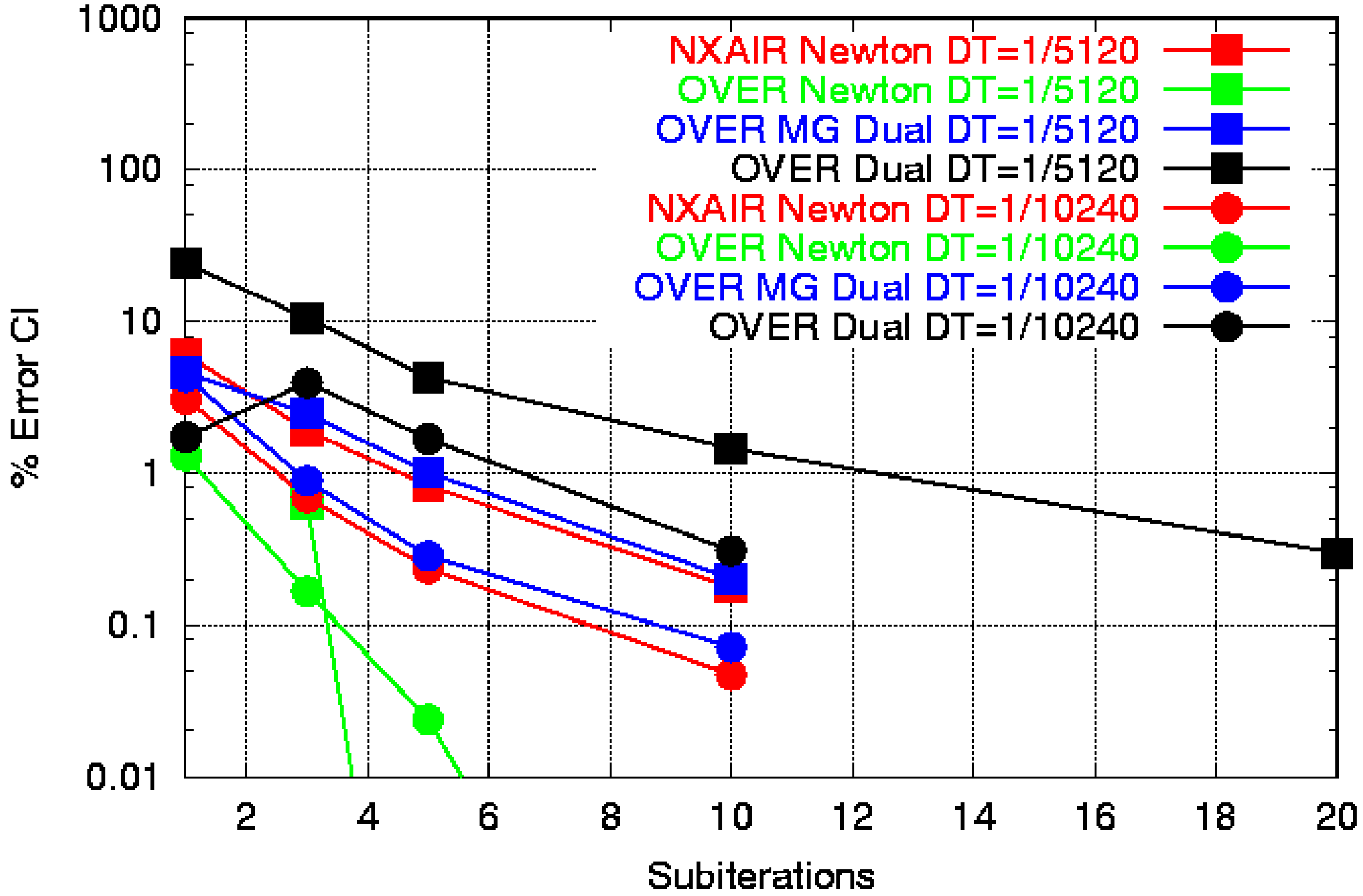




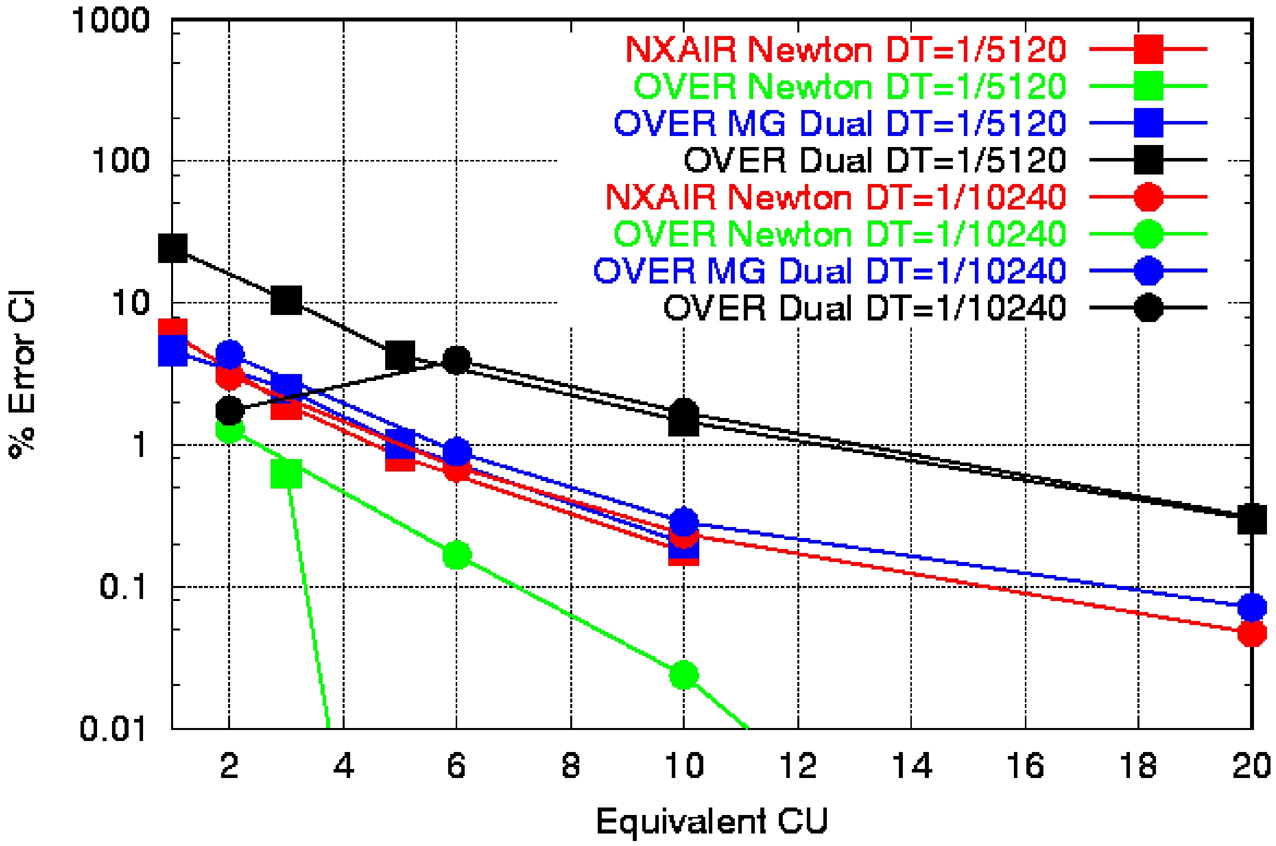




Pitching NACA0015 Airfoil



Pitching NACA0015 Airfoil Computational Cost



Summary

- Higher order time improves vortex convection performance for low numerical dissipation schemes
- GCL is required for deforming grids
- All algorithms demonstrated local convergence with increasing subiterations
- All algorithms showed improved or constant cost (CU) with increasing time step
- Need convergent inner algorithm